Example. There exist a Tychonoff space X and its point x_0 with a countable neighborhood base, and an open continuous map from X onto a regular T_1-space Y such that $\{f(x_0)\}$ is not a zero-set in Y.

Proof. Let Y be the Tychonoff cork screw or Mysior's example. The space Y is a regular T_1-space, and has a point y_0 with a countable neighborhood base $\{V_n : n \in \mathbb{N}\}$ such that

(1) $\mathrm{cl}\, V_{n+1} \subseteq V_n$ for each $n \in \mathbb{N}$,

(2) for each $n \in \mathbb{N}$, V_n contains no zero-set Z in Y with $y_0 \in Z$,

(3) every point $y \in Y \setminus \{y_0\}$ has a neighborhood base consisting of open-closed sets in Y.

Consider the subspace X_0 of $Y \times \{0, 1\}$ defined by

$$X_0 = \left\{ \langle y, 0 \rangle : y \in Y \setminus \bigcup_{n \in \mathbb{N}} (\mathrm{cl} V_{2n} \setminus V_{2n}) \right\} \cup \left\{ \langle y, 1 \rangle : y \in Y \setminus \bigcup_{n \in \mathbb{N}} (\mathrm{cl} V_{2n-1} \setminus V_{2n-1}) \right\}$$

Then, X_0 is a 0-dimensional T_1-space by (3), and hence, it is a Tychonoff space. Let X be the quotient space obtained from X_0 by collapsing the set $\{\langle y_0, 0 \rangle, \langle y_0, 1 \rangle\}$ to a point $x_0 \in X$. Then, X is also a 0-dimensional T_1-space, and hence, a Tychonoff space, and the point x_0 has a countable neighborhood base in X. Let $h : X_0 \to X$ be the quotient map and $\pi : X_0 \to Y$ the natural projection. Note that π is onto by (1). Then, there exists a map $f : X \to Y$ such that $f \circ h = \pi$. It is easily checked that f is an open continuous map onto Y, while $\{f(x_0)\}$ $(= \{y_0\})$ is not a zero-set in Y by (2). $\square$